Applying Boundary Conditions in the LightSolver PDE SDK

LightSolver Ltd. | Technical Whitepaper

Abstract

This whitepaper describes the boundary-condition mapping used by the LightSolver PDE toolbox. The physical operator uses a one-cell frame of retained reference variables, while the optical round-trip operator uses two apparatus layers: an inner port ring on the wall faces and an outer port ring. These optical port rows implement Robin conditions as contractive phase relations that interpolate continuously between Dirichlet and Neumann limits.

1. Boundary-condition specification

1.1 Unified Robin condition

On each boundary face, the toolbox accepts the Robin data

$$\alpha_j\, u_{\beta,j} + \beta_j\, q_{n,j} = \gamma_j, \quad (1)$$

where $u_{\beta,j}$ is the field value on the wall face and $q_{n,j}$ is the outward normal flux density. The sign convention is positive for flux leaving the domain. Dirichlet, Neumann, and mixed conditions remain parameter choices within this single form.

Type	α	β	γ	Meaning
Dirichlet	1	0	u^D	Prescribed wall value
Neumann	0	1	q^N	Prescribed outward flux
Robin	α	β	γ	Mixed value and flux

2. Boundary architecture

The boundary implementation is organized into three layers, as illustrated in Figure 1: the **inner port row**, the **outer port row**, and the **reference ring**. The inner port row is the outermost row of the core computational grid, while the outer port row lies immediately outside it. The reference ring forms the outermost layer, and provides a common phase reference. This terminology is used throughout the remainder of the document.

2.1 Physical operator L, Optical operator T

The finite-volume discretization is written in the linear-system form

$$Lu = R, \quad (2)$$

where u is the discretized field, L is the physical operator containing the finite-volume stencil coefficients, and R contains the corresponding source and prescribed boundary contributions.

The physical operator L contains the core computational grid together with one additional **outer port row** on each side. The **inner port row** is the core row located directly at the boundary of the computational domain and therefore remains part of the standard core grid. Each core cell keeps its ordinary 3 × 3 balance row, including couplings that land on the outer port row. The outer port row lies immediately outside the inner port row, its nodes are retained unknowns in L.

Consider the north boundary at column j. The boundary face lies between the inner and outer port rows. The field value at the face is represented by the midpoint between the outer-port value u_j^r and the corresponding inner-port value:

$$u_{b,j} = \frac{1}{2}\left(u_j^r + u_{N_y-1,j}\right).$$

The outward normal flux $q_{n,j}$ is evaluated using the same north-face finite-volume discretization used by the interior operator. Together, the boundary field and normal flux define the Robin equation associated with each outer-port location as will be shown in section 3.

2.2 Corners

The four padded physical corners do not own wall faces and therefore receive no boundary condition equation. They are closed by bilinear extrapolation

$$u_{corner} = u_a + u_b - u_{diag}, \quad R_{corner} = 0,$$

where u_a and u_b are the two neighboring frame values and u_{diag} is the caller cell diagonally inside. This closure preserves constants and fields that are linear in the two coordinates, avoiding an artificial dependence on the absolute field offset.

2.3 Optical operator T

The optical operator T extends the same structure by adding the **reference ring** outside the outer port row. The resulting architecture is summarized in the table below:

Node class	Location	Role in T	Readout / residual
Caller cells	Inside port ring	Physical operator and source	Included
Port ring	On wall faces	One Robin phase relation per face	Excluded
Reference ring	Outermost layer	Phase-locked anchor chain	Excluded

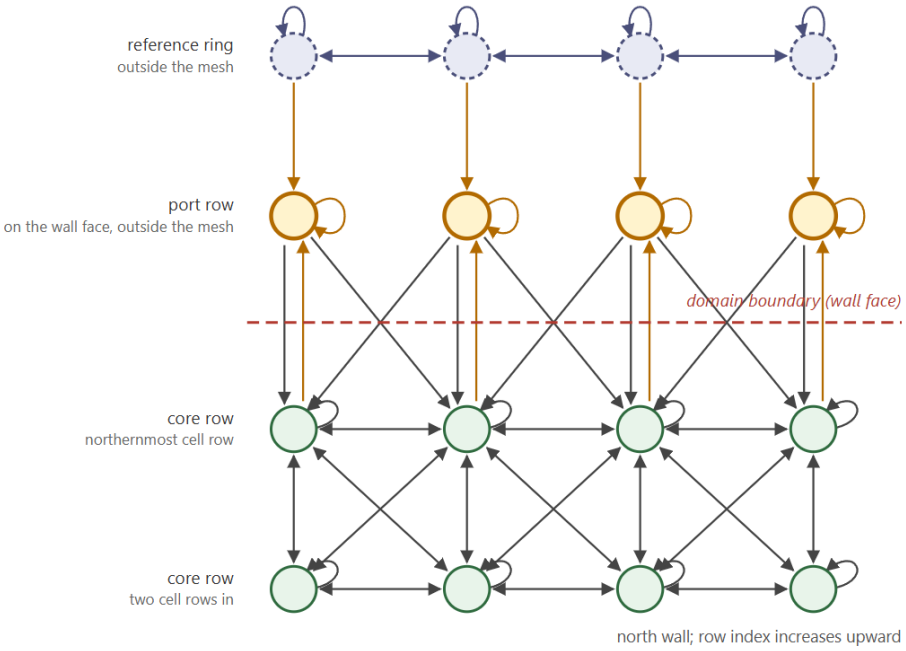


Figure 1: Optical implementation of boundary conditions

3. Robin boundary-condition mapping

The boundary architecture introduced in Section 2 provides a direct representation of the Robin condition (1). For each boundary location, the **inner port row** is the boundary row of the core computational grid, the **outer port row** represents the corresponding wall face, and the **reference ring** provides the common phase reference used by the optical representation.

We denote the two orthogonal coordinates of the computational grid by ξ and η. Unlike a uniform Cartesian grid, the spacing along these coordinates is not necessarily constant; therefore, the local grid spacings are denoted by $\Delta\xi$ and $\Delta\eta$.

3.1 Optical mapping of the Robin condition

In the optical operator, each outer-port node represents the corresponding Robin boundary relation through two principal couplings. The **outward coupling to the reference ring** represents the field contribution, while the **inward coupling to the inner port row** represents the normal-flux contribution. The inward coupling spans the half-cell distance between the wall face and the center of the corresponding inner-port cell.

To characterize the relative contributions, define the half-cell compliance and conductance as

$$\lambda_j = \frac{\Delta\eta_{N_y-1}}{2K_{\eta\eta,N_y-1,j}},\ \kappa_j = \frac{1}{\lambda_j},$$

Where $\widehat{K}_{\eta\eta,N_y-1,j}$ is the effective normal conductivity evaluated on the north boundary face at column j.

The relative field participation is then defined as

$$\tau_j = \frac{|\ \alpha_j\ |}{|\ \alpha_j\ | + |\ \beta_j\ |\ \kappa_j}.$$

With α_j and β_j as defined in equation (1).

The outward leg to the **reference ring** receives weight τ_j, while the inward leg to the **inner port row** receives weight $1 - \tau_j$. Thus,

$$\tau_j \to 1$$

corresponds to the Dirichlet limit, whereas

$$\tau_j \to 0$$

corresponds to the Neumann limit. Intermediate values represent mixed Robin conditions.

The corresponding formulations for the remaining faces follow analogously and are omitted for brevity.

3.2 Anisotropic cross terms

For a skewed or anisotropic discretization, the normal flux may also contain a tangential contribution,

$$q_n = \widehat{K}_{\eta\eta} \frac{\partial u}{\partial \eta} + \widehat{K}_{\eta\xi} \frac{\partial u}{\partial \xi}.$$

In this case, the outer-port row additionally couples to the two diagonally adjacent nodes of the inner port row. These two corner coefficients have equal magnitude and opposite sign. Wall-parallel east and west coefficients are zero for a north-boundary outer-port row. When $K_{\eta\xi} = 0$, the corner couplings vanish and the mapping reduces to the two principal couplings between the outer port and the reference ring and inner port row.

The center coefficient is chosen so that the signed sum of the flux-stencil coefficients is zero. Consequently, the discrete flux contribution vanishes for a spatially constant field.

3.3 Affine boundary data and the reference phase

As introduced in equation (2), the discretized problem is written as $Lu = R$. For the Robin boundary condition, α_j and β_j determine the coefficients of the boundary row in L, while the prescribed value γ_j contributes to the corresponding boundary entry in R.

The optical phases represent the field relative to a common phase reference established by the reference ring. The physical field is recovered through the affine mapping $u = U\phi + u_{\mathrm{ref}}$, where u_{ref} provides the additive field level needed to satisfy the prescribed boundary data γ_j.

The pure-Neumann case is different. If $\alpha_j = 0$ on all boundary faces, the boundary conditions prescribe only the normal flux and contain no field-value contribution. They therefore do not fix the additive constant, or absolute field level, of the solution. Consequently, u_{ref} cannot be determined from the boundary conditions alone.

4. Assembly and solve sequence

1. **Assemble caller rows:** Build the usual finite-volume 3 × 3 rows and source terms on caller cells.
2. **Add the physical frame:** Retain one frame of reference variables and place one integrated Robin equation on each wall-face node of L.

3. **Close corners:** Apply bilinear extrapolation at the four physical-frame corners.
4. **Scale physical rows:** Apply physical-operator scaling after the outer caller layers have been retargeted to their half-cell wall geometry. Reference rows are excluded from the physical normalization measure.
5. **Build optical apparatus:** Pad T by two layers, overwrite the inner layer with port rows, and overwrite the outer layer with the phase-locked reference-ring rows.
6. **Solve and read out:** Evolve the caller, port, and reference lasers, then discard both apparatus layers and subtract the reference-ring mean before converting phase to field.

For more information, see:

Tradonsky C., Wolf O., Vaknin T., Gluck D. & Furman D., “Solving Partial Differential Equations on an Analog, Optical Platform,” ACM Computing Frontiers, 2025, pp. 130–134.